

STUDY OF HEAVY-ION ELASTIC SCATTERING WITHIN CLASSICAL AND QUANTUM OPTICAL MODEL

Introduction

Elastic scattering is the main reaction channel of heavy-ion collisions at all incident energies and, thus, it is a useful experimental tool for study of nuclear structure and nucleus-nucleus interaction potential. In fact, elastic scattering of nuclear particles is a rather complicate process. Relative motion of nuclei in the elastic scattering channel (when both nuclei remain in their ground states) is strongly coupled to other degrees of freedom (nucleon motion inside colliding nuclei, deformations of nuclear surfaces and rotation of nuclei). At above barrier incident energies a considerable part of incoming flux goes away from the elastic scattering into the other reaction channels. This means that the elastic scattering data give us also some information on the other reaction channels and on collision dynamics in whole.

For all nuclear particles, with the exception of neutrons, the Coulomb repulsive interaction dominates at large distances: $V(r \rightarrow \infty) = Z_1 Z_2 e^2 / r$. When nuclei approach each other, the attractive nuclear forces have an effect. Resulting nucleus-nucleus interaction potential $V(r)$ looks as shown in Fig. 1. It has a barrier V^B located at distance $R^B > R_1 + R_2$, where R_i is the radius of i -th nucleus. Precise value of the nucleus-nucleus potential $V(r)$ is not known, and careful analysis of experimental data on elastic scattering of two nuclei just allows one to determine their interaction potential. This nucleus-nucleus potential can be applied afterwards to study the other reaction channels.

Any excitation of colliding nuclei takes out them from the elastic channel. Such decreasing of incoming flux (number of elastically scattered particles) can be simulated by additional imaginary (absorptive) potential $iW(r)$, $W(r) < 0$ and $W(r \rightarrow \infty) \rightarrow 0$. This potential defines the mean free path of relative motion of nuclei: $\lambda_{free} = -\hbar v / 2W(r)$, which, in turn, defines the probability for the nuclear system to remain in the elastic channel.

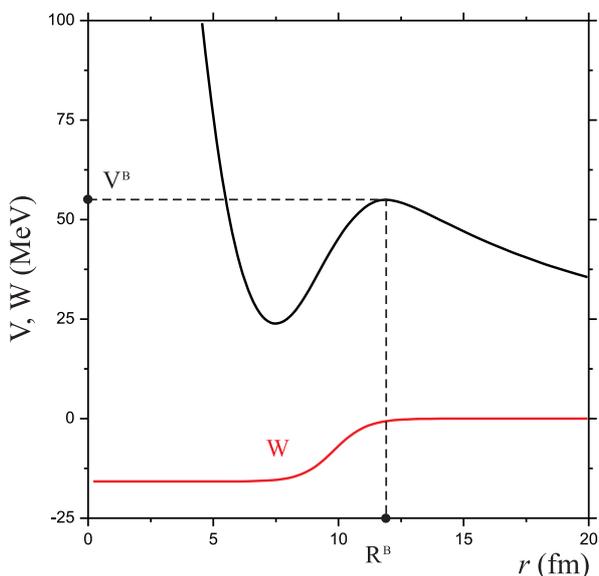


Fig. 1. Interaction potential of ^{12}C with ^{208}Pb calculated within the proximity model (real part) and schematic view of the absorption potential $W(r)$.

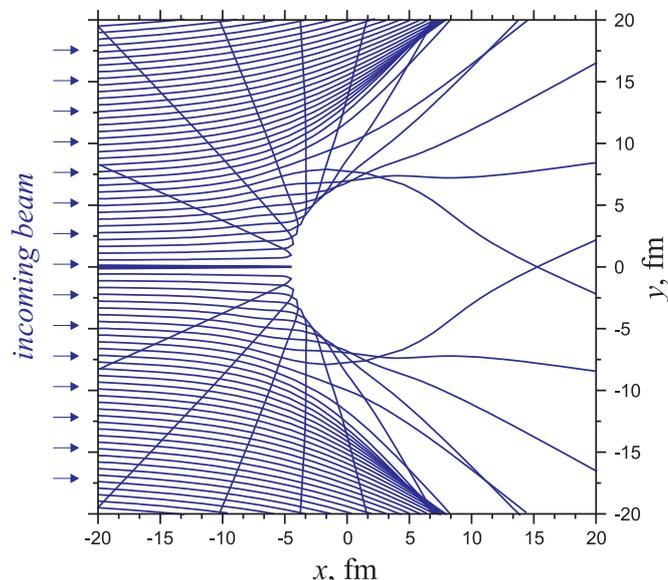


Fig. 2. Classical trajectories, calculated within classical model of nuclear scattering for the $^{12}\text{C} + ^{208}\text{Pb}$ elastic scattering at laboratory beam energy of 100 MeV.

In the course of the project student will analyze experimental data on elastic scattering for a given combination of nuclei (projectile + target). From the calculated angular distribution of elastic scattering he will determine nucleus-nucleus potential (its real and imaginary parts), find classical tra-

jectories and deflection function, plot the partial wave functions and partial S-matrix elements, view and interpret three-dimensional wave function and so on.

1. Elastic scattering in classical dynamics model

For heavy ion collisions the de-Broglie wavelength of their relative motion is usually rather small, less than $R_1 + R_2$, and $k(R_1 + R_2) \gg 1$, where $k = \sqrt{2\mu E} / \hbar$ is the wavenumber. In this case the properties of the corresponding wave function are defined mainly by the classical trajectories, and collision dynamics can be understood much better and quite clear just within the classical model. For nucleon-nucleus scattering this relation is justified at energies $E_{p,n} > 80$ MeV, for alpha-particles at $E_\alpha > 20$ MeV and for heavier ions at any above barrier energy.

Classical model does not allow one to describe quantum effects such as interference pattern in angular distributions of reaction products, which is a result of wave nature of colliding nuclear particles. Nevertheless the classical model is intuitively better understandable since it operates with such classical conceptions as a trajectory, impact parameter, deflection angle and so on (see Fig. 3). Therefore, a joint analysis of scattering processes within quantum and classical models can provide us a deep understanding of heavy-ion collision dynamics.

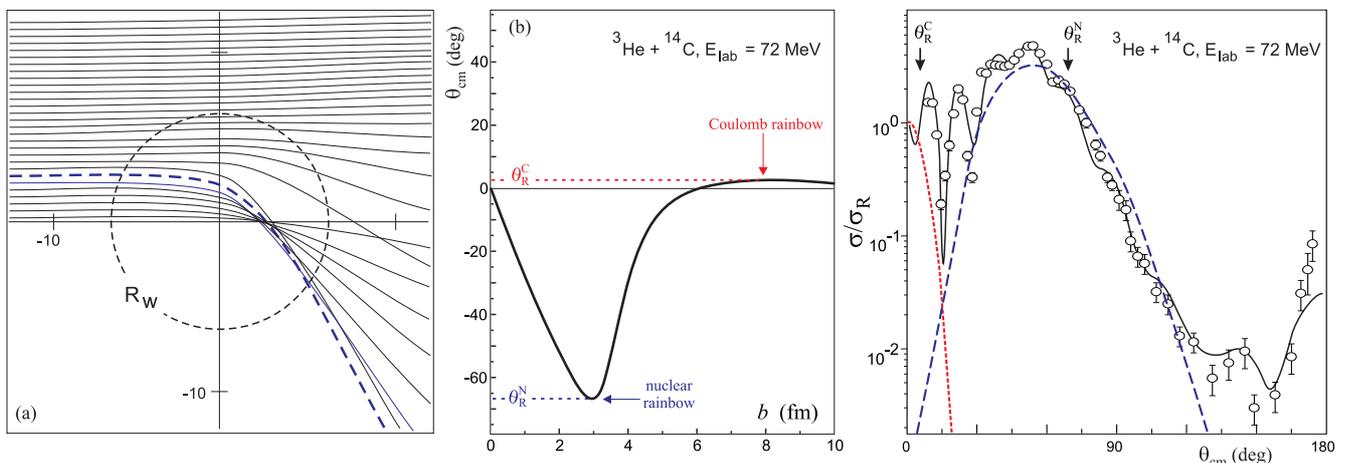


Fig. 3. (a) Classical trajectories of ${}^3\text{He}$ projectile scattered by ${}^{14}\text{C}$ target at beam energy of 24 MeV/nucleon. Dashed line corresponds to the nuclear rainbow impact parameter. The absorptive region (non-zero values of imaginary part of the optical potential) is shown by dashed circle. (b) Classical deflection function. Coulomb ($\theta_R^C \approx +2^\circ$) and nuclear ($\theta_R^N \approx -67^\circ$) rainbow angles are shown. (c) Differential cross section of ${}^3\text{He}$ elastic scattering by ${}^{14}\text{C}$ at energy 24 MeV/u. Dotted and dashed curves show the contribution of the Coulomb and nuclear rainbow. Solid curve is the result of quantum calculation performed within the optical model.

In the course of practical training work, student will study the classical model of elastic scattering and will perform the corresponding trajectory analysis. It includes (1) calculation of the scattering trajectories (Fig. 3a), (2) analysis of the properties of a deflection function (Fig. 3b), (3) study of nuclear and Coulomb rainbow phenomena, (4) calculation of the differential cross section (angular distribution) of elastic scattering within semiclassical approximation (Fig. 3c), and others. All these problems will be solved with the use of the low-energy nuclear knowledge base (NRV) located at the website <http://nrv.jinr.ru/nrv>. The NRV system is designed to be used in the Internet environment. Access to its resources is available to any user via any web-browser (see Fig. 4). The results of calculations are plotted using the Java-applets in separate windows and can be saved (downloaded) on a user's machine in graphical (GIF, EPS, etc.) or text format.

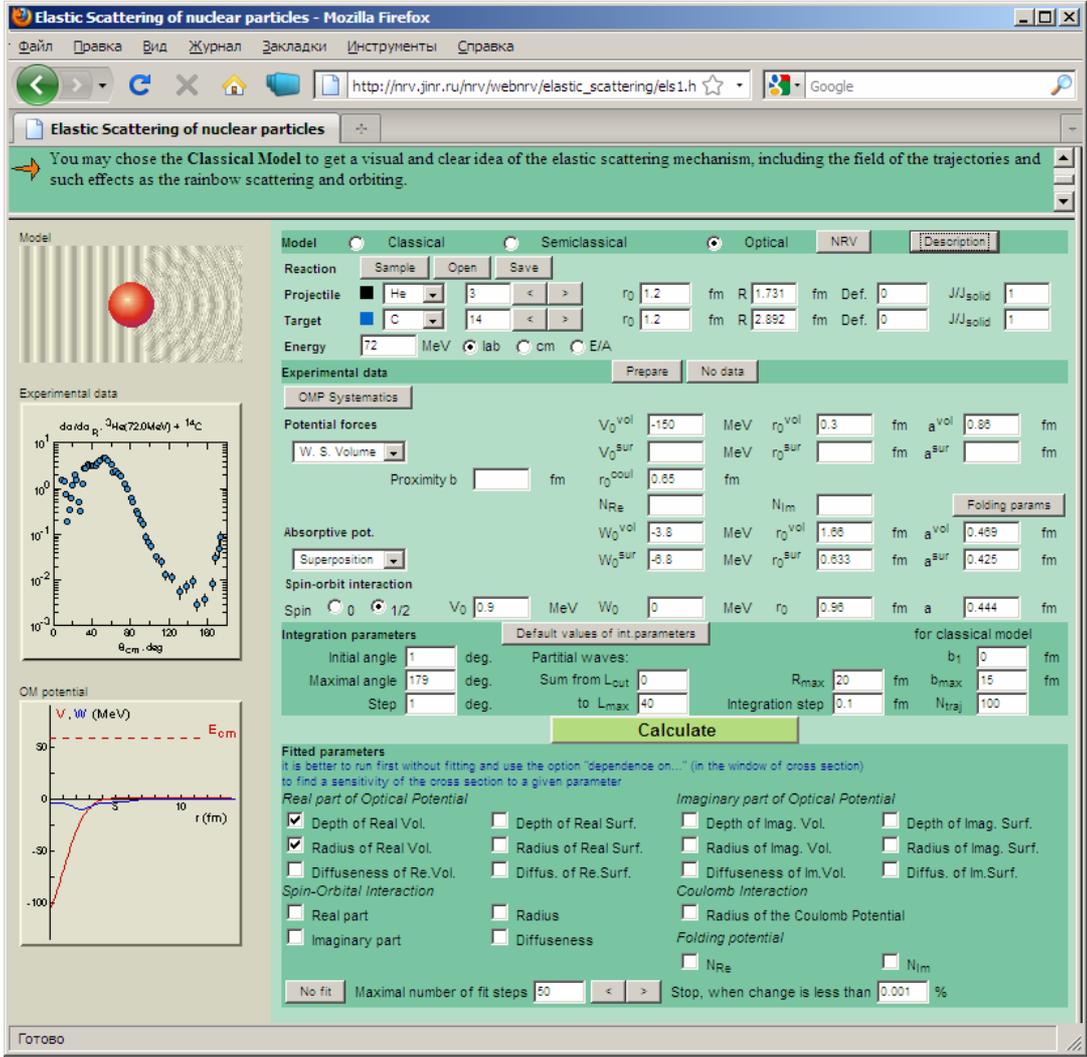


Fig. 4. Web-dialog window used for preparation of initial data and starting the calculation of elastic scattering cross section within classical and quantum optical models.

2. Quantum Optical Model of elastic scattering

The optical model of elastic scattering was proposed for the first time by H. Feshbach, K. Porter and V. F. Weisskopf. The corresponding theory basing on projection operator technique was developed later by Herman Feshbach. The main point of the model consists in introduction of a complex nucleus-nucleus potential $V(r) + iW(r)$ (called optical model potential) which describes a relative motion of colliding particles in elastic channel and a decrease of the outgoing flux due to coupling with inelastic reaction channels (it is provided by negatively defined imaginary part $W < 0$). It is assumed that an influence of all these reaction channels on the elastic scattering can be simulated by an appropriate choice of the optical potential. Thus, in quantum mechanical optical model of elastic scattering the relative motion of the projectile and target nuclei is described by an effective one-body Schrödinger equation

$$\left[-\frac{\hbar^2}{2\mu} \nabla^2 + V_{OM}(r) \right] \Psi_k^{(+)}(\vec{r}) = E \Psi_k^{(+)}(\vec{r}), \quad (1)$$

where $E = \hbar^2 k^2 / 2\mu$ is the relative motion energy, μ is the reduced mass, and $V_{OM}(r)$ is the phenomenological optical potential. Simplicity of the optical model allows one to fit the optical potential parameters (by χ^2 minimization procedure) in order to describe corresponding experimental data (see,

for example, Fig. 5).

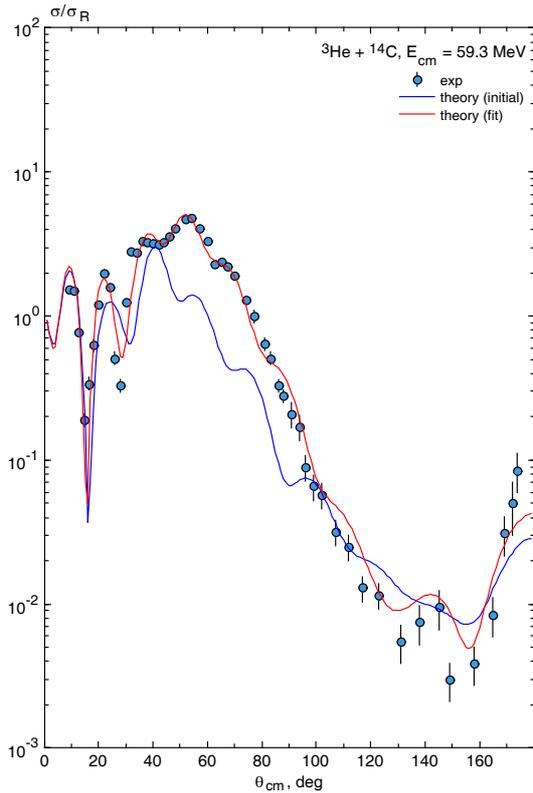


Fig. 5. Comparison of the experimental (dots) and optical model (curves) elastic scattering angular distributions for the ${}^3\text{He} + {}^{14}\text{C}$ collisions at $E_{\text{lab}} = 72$ MeV. Calculations were performed with some initial set of parameters of optical potential (blue curve) and after automatic fit of these parameters (red curve).

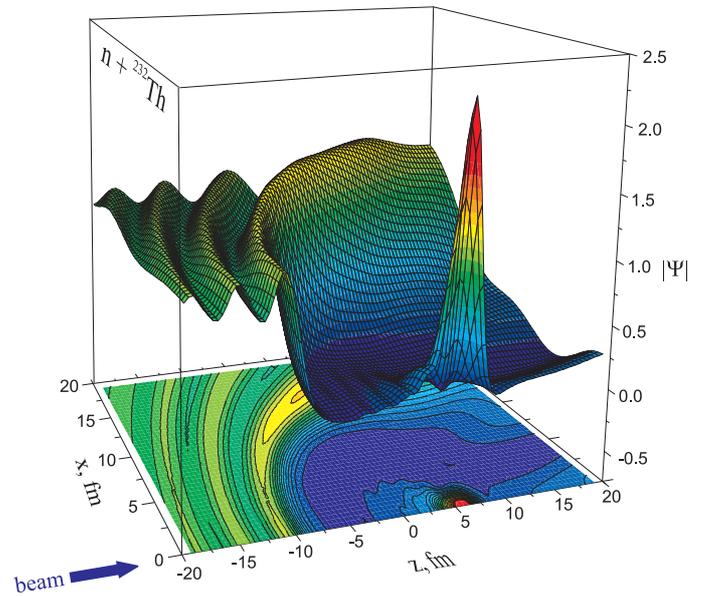


Fig. 6. Three-dimensional plot of the wave function describing elastic scattering of $n + {}^{232}\text{Th}$ at $E_n = 14$ MeV.

The optical model turned out to be the effective tool for description of heavy-ion elastic scattering which in addition provides important information on nucleus-nucleus interaction. Obtained optical potential can afterwards be applied to study the other reaction channels such as inelastic scattering, transfer reactions, deep-inelastic scattering, fusion and fusion-fission, and so on.

In the course of the project student has a chance to study the optical model and get practical skills of its use for analysis of experimental data. It includes (1) studying the partial decomposition of the total scattering wave function, (2) calculation of the scattering amplitude and quantum differential cross section, (3) choice of an appropriate shape of phenomenological optical potential, (4) studying the dependence of angular distribution on optical potential parameters, (5) knowledge of numerical methods for solution of the Schrödinger equation, (6) an idea on the properties of the partial wave functions and phase-shifts, (7) imaging the three-dimensional wave function (see Fig. 6), and many others.

Literature:

1. G.R. Satchler, *Direct Nuclear Reactions*, Clarendon Press, 1983.
2. N.K. Glendenning, *Direct Nuclear Reactions*, Academic Press, 1983.
3. P. Frobrich and R. Lipperheide, *Theory of nuclear reactions*, Clarendon Press, Oxford, 1996.
4. G.R. Satchler and W.G. Love, *Phys. Rep.*, 1979, **55**, 183 – 254.
5. M.E. Brandan and G.R. Satchler, The interaction between light heavy-ions and what it tells us, *Phys. Rep.*, 1997, **285**, 143 – 243.